Chapter 14

Fluids

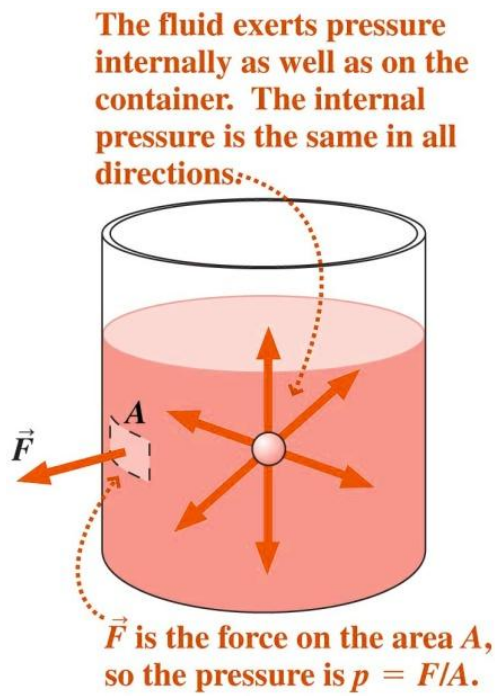
14.2 What is a Fluid?

- **Fluid**: Matter that flows under the influence of external forces
- Includes gases and liquids:
  - In gases, molecules are far apart and the density changes readily.
  - In liquids, molecules are close together and density remains essentially constant.
- Fluids conform to the boundaries of and assume the configuration of any container they are placed in:
  - A fluid cannot sustain a force that is tangential to its surface -- cannot withstand a shearing stress
  - Can exert a force in the direction perpendicular to surface
  - Cannot maintain a fixed structure

14.3 Density & Pressure

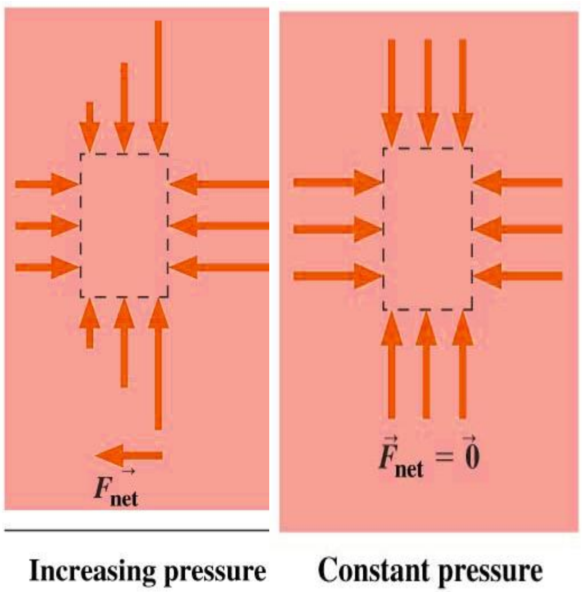
- To find the density ρ of a fluid at any point, isolate a small volume element V around that point and measure the mass m of the fluid contained within that element. If the fluid has uniform density, then:

$$\boxed{\rho = \frac{\Delta m}{\Delta V} = \frac{m}{V}}$$



- Density is a scalar property
- SI unit: kilogram per cubic meter (kg/m^3)
  - gram per cubic centimeter (g/cm^3);
  - 1 cm^3 = 1 cc = 1 mL
- If the normal force exerted over a flat area A is uniform over that area, then the pressure is defined as:

$$\boxed{p = \frac{F}{A}}$$



- SI unit: newton per square meter; pascal (Pa): 1 Pa = 1 N/m^2
- *1 atmosphere (atm) = 1.01×10^5 Pa = 101 kPa = 1013 mB = 760 mm Hg = 760 torr = 14.7 lb/in^2 (psi)*

14.3 Density & Pressure

Table 14-1

Some Densities

Material or Object	Density (kg/m^3)	Material or Object	Density (kg/m^3)
Interstellar space	10^{-20}	Iron	7.9×10^3
Best laboratory vacuum	10^{-17}	Mercury (the metal, not the planet)	13.6×10^3
Air: 20°C and 1 atm pressure	1.21	Earth: average	5.5×10^3
20°C and 50 atm	60.5	core	9.5×10^3
Styrofoam	1×10^2	crust	2.8×10^3
Ice	0.917×10^3	Sun: average	1.4×10^3
Water: 20°C and 1 atm	0.998×10^3	core	1.6×10^5
20°C and 50 atm	1.000×10^3	White dwarf star (core)	10^{10}
Seawater: 20°C and 1 atm	1.024×10^3	Uranium nucleus	3×10^{17}
Whole blood	1.060×10^3	Neutron star (core)	10^{18}

Table 14-2

Some Pressures

	Pressure (Pa)		Pressure (Pa)
Center of the Sun	2×10^{16}	Automobile tire[a]	2×10^5
Center of Earth	4×10^{11}	Atmosphere at sea level	1.0×10^5
Highest sustained laboratory pressure	1.5×10^{10}	Normal blood systolic pressure[a,b]	1.6×10^4
Deepest ocean trench (bottom)	1.1×10^8	Best laboratory vacuum	10^{-12}
Spike heels on a dance floor	10^6		

[a]Pressure in excess of atmospheric pressure. [b]Equivalent to 120 torr on the physician's pressure gauge.

Example: Atmospheric Pressure & Force

A living room has floor dimensions of 3.5 m and 4.2 m and a height of 2.4 m.

(a) What does the air in the room weigh when the air pressure is 1.0 atm?

KEY IDEAS

(1) The air's weight is equal to mg, where m is its mass. (2) Mass m is related to the air density ρ and the air volume V by Eq. 14-2 ($\rho = m/V$).

Calculation: Putting the two ideas together and taking the density of air at 1.0 atm from Table 14-1, we find

$$\begin{aligned} mg &= (\rho V)g \\ &= (1.21\ \mathrm{kg/m^3})(3.5\ \mathrm{m} \times 4.2\ \mathrm{m} \times 2.4\ \mathrm{m})(9.8\ \mathrm{m/s^2}) \\ &= 418\ \mathrm{N} \approx 420\ \mathrm{N}. \qquad \text{(Answer)} \end{aligned}$$

This is the weight of about 110 cans of Pepsi.

(b) What is the magnitude of the atmosphere's downward force on the top of your head, which we take to have an area of 0.040 m^2?

KEY IDEA

When the fluid pressure p on a surface of area A is uniform, the fluid force on the surface can be obtained from Eq. 14-4 ($p = F/A$).

Calculation: Although air pressure varies daily, we can approximate that $p = 1.0$ atm. Then Eq. 14-4 gives

$$\begin{aligned} F = pA &= (1.0\ \mathrm{atm})\left(\frac{1.01 \times 10^5\ \mathrm{N/m^2}}{1.0\ \mathrm{atm}}\right)(0.040\ \mathrm{m^2}) \\ &= 4.0 \times 10^3\ \mathrm{N}. \qquad \text{(Answer)} \end{aligned}$$

This large force is equal to the weight of the air column from the top of your head to the top of the atmosphere.

14.4: Fluids at Rest (fluid statics/hydrostatics)

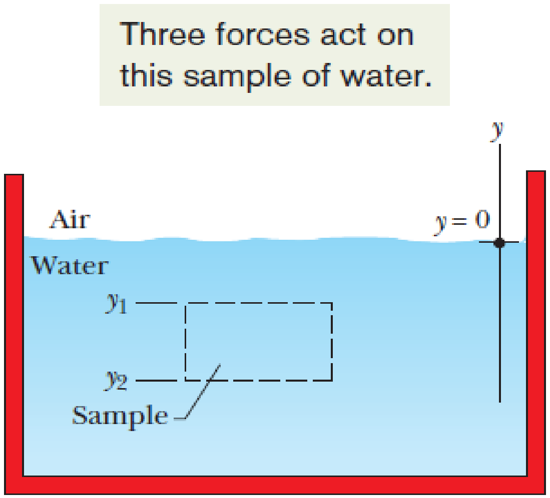
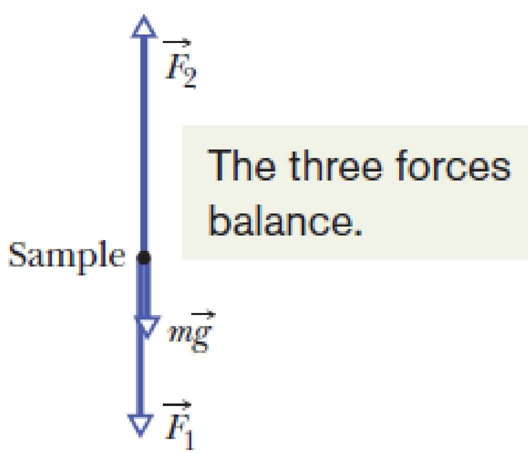


Fig. 14-2 Above: A tank of water in which a sample of water is contained in an imaginary cylinder of horizontal base area A.

Below: A free-body diagram of the water sample.



The pressure at a point in a fluid in hydrostatic equilibrium depends on the depth of that point but not on any horizontal dimension of the fluid or its container.

The balance of the 3 forces is written as: $F_2 = F_1 + mg.$

If p_1 and p_2 are the pressures on the top and the bottom surfaces of the sample:

$$F_1 = p_1 A \quad \text{and} \quad F_2 = p_2 A.$$

Since the mass m of the water in the cylinder is, $m = \rho V$, where the cylinder's volume V is the product of its face area A and its height *(y_1 - y_2), then* $m = \rho A(y_1 - y_2)$.

Therefore, $p_2 A = p_1 A + \rho A g(y_1 - y_2)$

$$p_2 = p_1 + \rho g(y_1 - y_2).$$

If y_1 is at the surface and y_2 is at a depth h below the surface, then:

$$\boxed{p = p_0 + \rho g h}$$

(where p_o is the pressure at the surface, and p the pressure at depth h).

Example:

Gauge pressure on a scuba diver

A novice scuba diver practicing in a swimming pool takes enough air from his tank to fully expand his lungs before abandoning the tank at depth L and swimming to the surface. He ignores instructions and fails to exhale during his ascent. When he reaches the surface, the difference between the external pressure on him and the air pressure in his lungs is 9.3 kPa. From what depth does he start? What potentially lethal danger does he face?

KEY IDEA

The pressure at depth h in a liquid of density ρ is given by Eq. 14-8 ($p = p_0 + \rho gh$), where the gauge pressure ρgh is added to the atmospheric pressure p_0.

Calculations: Here, when the diver fills his lungs at depth L, the external pressure on him (and thus the air pressure within his lungs) is greater than normal and given by Eq. 14-8 as

$$p = p_0 + \rho gL,$$

where p_0 is atmospheric pressure and ρ is the water's density (998 kg/m^3, from Table 14-1). As he ascends, the external pressure on him decreases, until it is atmospheric pressure p_0 at the surface. His blood pressure also decreases, until it is normal. However, because he does not exhale, the air pressure in his lungs remains at the value it had at depth L. At the surface, the pressure difference between the higher pressure in his lungs and the lower pressure on his chest is

$$\Delta p = p - p_0 = \rho gL,$$

from which we find

$$L = \frac{\Delta p}{\rho g} = \frac{9300 \text{ Pa}}{(998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)}$$

$$= 0.95 \text{ m.} \qquad \text{(Answer)}$$

This is not deep! Yet, the pressure difference of 9.3 kPa (about 9% of atmospheric pressure) is sufficient to rupture the diver's lungs and force air from them into the depressurized blood, which then carries the air to the heart, killing the diver. If the diver follows instructions and gradually exhales as he ascends, he allows the pressure in his lungs to equalize with the external pressure, and then there is no danger.

Example:

The U-tube in Fig. 14-4 contains two liquids in static equilibrium: Water of density ρ_w (= 998 kg/m^3) is in the right arm, and oil of unknown density ρ_x is in the left. Measurement gives l = 135 mm and d = 12.3 mm. What is the density of the oil?

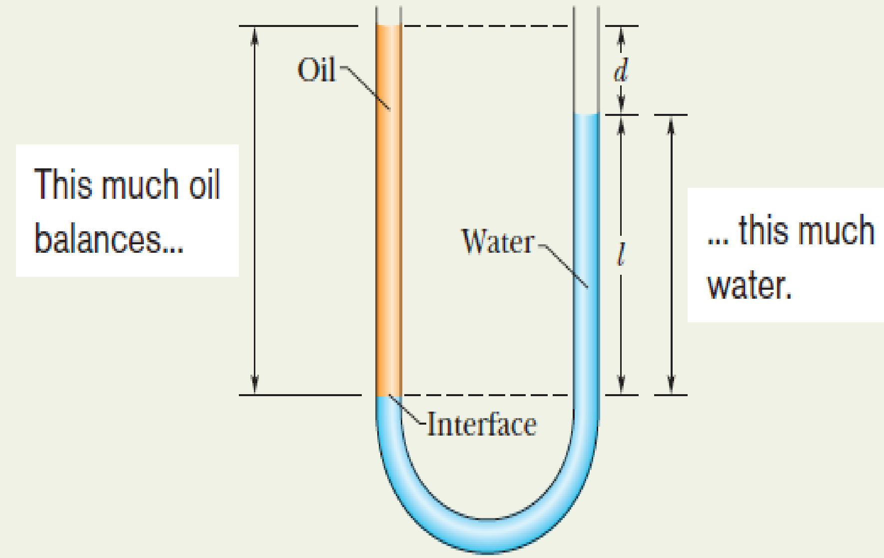


Fig. 14-4 The oil in the left arm stands higher than the water in the right arm because the oil is less dense than the water. Both fluid columns produce the same pressure p_{int} at the level of the interface.

KEY IDEAS

(1) The pressure p_{int} at the level of the oil–water interface in the left arm depends on the density ρ_x and height of the oil above the interface. (2) The water in the right arm *at the same level* must be at the same pressure p_{int}. The reason is that, because the water is in static equilibrium, pressures at points in the water at the same level must be the same even if the points are separated horizontally.

Calculations: In the right arm, the interface is a distance l below the free surface of the *water*, and we have, from Eq. 14-8,

$$p_{int} = p_0 + \rho_w g l \quad \text{(right arm)}.$$

In the left arm, the interface is a distance $l + d$ below the free surface of the *oil*, and we have, again from Eq. 14-8,

$$p_{int} = p_0 + \rho_x g(l + d) \quad \text{(left arm)}.$$

Equating these two expressions and solving for the unknown density yield

$$\rho_x = \rho_w \frac{l}{l + d} = (998\ \text{kg/m}^3)\frac{135\ \text{mm}}{135\ \text{mm} + 12.3\ \text{mm}}$$

$$= 915\ \text{kg/m}^3. \qquad \text{(Answer)}$$

Note that the answer does not depend on the atmospheric pressure p_0 or the free-fall acceleration g.

14.5: Measuring Pressure: The Mercury Barometer

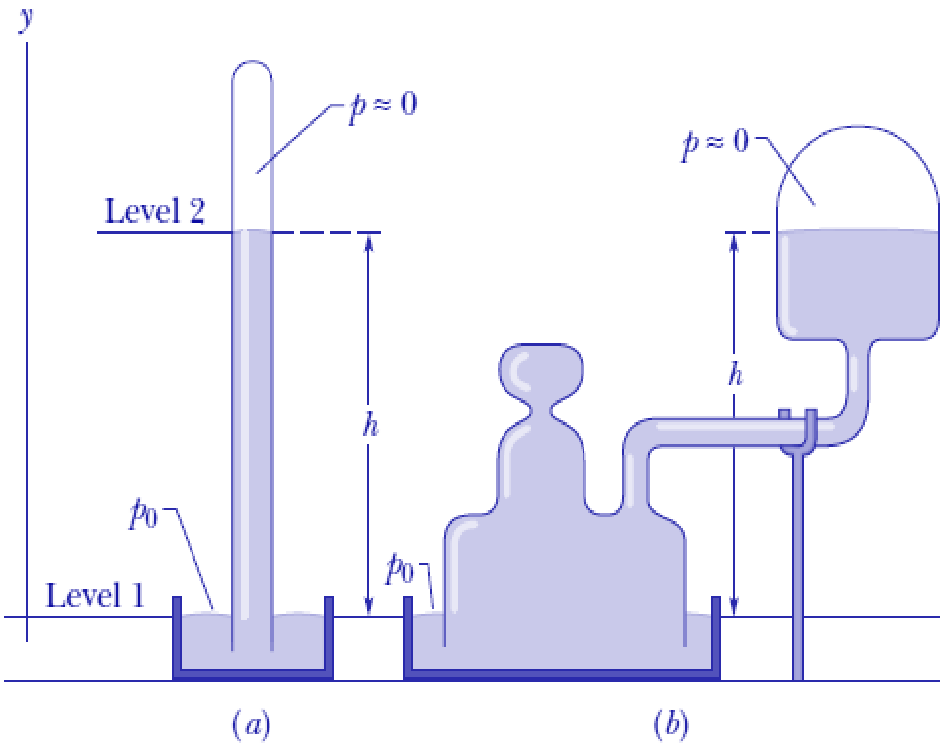


Fig. 14-5 (a) A mercury barometer. (b) Another mercury barometer. The distance h is the same in both cases.

- A Hg barometer is a device used to measure the pressure of the atmosphere.
- The long glass tube is filled with Hg and the space above the Hg column contains only Hg vapor, whose pressure can be neglected.
- If the atmospheric pressure is p_0, and ρ is the density of Hg:

$$\boxed{p_0 = \rho g h,}$$

14.5: Measuring Pressure: The Open-Tube Manometer

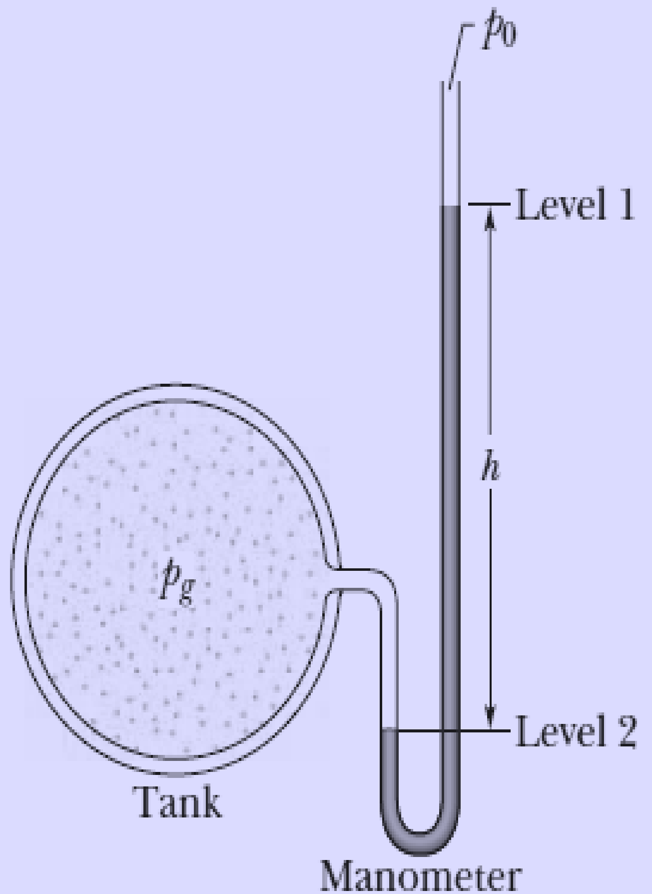


Fig. 14-6 An open-tube manometer, connected to measure the gauge pressure of the gas in the tank on the left. The right arm of the U-tube is open to the atmosphere.

- An open-tube manometer measures the gauge pressure p_g of a gas.
- Consists of a U-tube containing a liquid, with one end of the tube connected to the vessel whose gauge pressure is to measured and the other end open to the atmosphere.
- If p_o is the atmospheric pressure, p is the pressure at level 2 as shown, and ρ is the density of the liquid in the tube, then:

$$\boxed{p_g = p - p_0 = \rho g h,}$$

14.6: Pascal's Principle

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

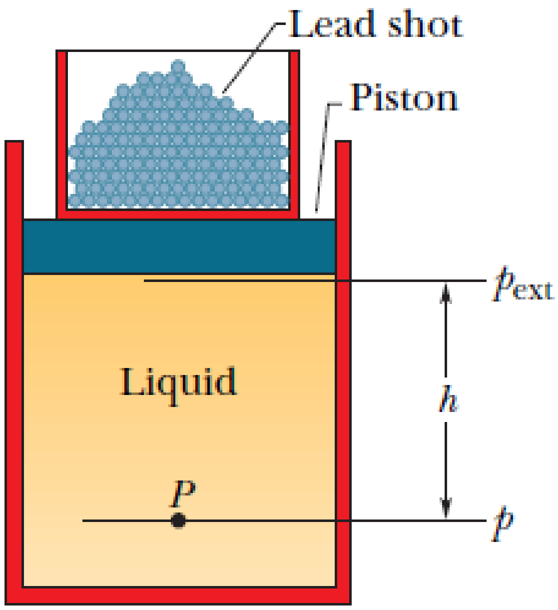


Fig. 14-7 Lead shot (small balls of lead) loaded onto the piston create a pressure p_{ext} at the top of the enclosed (incompressible) liquid. If p_{ext} is increased, by adding more lead shot, the pressure increases by the same amount at all points within the liquid.

14.6: Pascal's Principle & the Hydraulic Lever

- The force F_i is applied on the left and the downward force F_o from the load on the right produce a change Δp in the pressure of the liquid that is given by:

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o},$$

$$F_o = F_i \frac{A_o}{A_i}.$$

- If the input piston is moved downward a distance d_i, the output piston moves upward a distance d_o, such that the same volume V of the incompressible liquid is displaced at both pistons.

$$V = A_i d_i = A_o d_o,$$

$$d_o = d_i \frac{A_i}{A_o}.$$

- Then the output work is:

$$W = F_o d_o = \left(F_i \frac{A_o}{A_i}\right)\left(d_i \frac{A_i}{A_o}\right) = F_i d_i$$

... a large output force.

A small input force produces ...

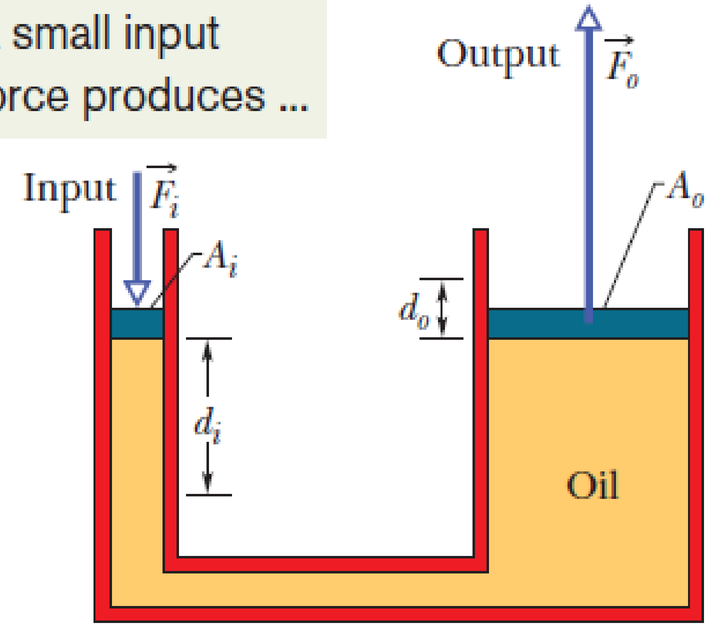


Fig. 14-8 A hydraulic arrangement that can be used to magnify a force $\vec{F}_i$. The work done is, however, not magnified and is the same for both the input and output forces.

14.7: Archimedes Principle

- When a body is fully or partially submerged in a fluid, a buoyant force from the surrounding fluid acts on the body.

- The force is directed upward and has a magnitude equal to the weight of the fluid that has been displaced by the body.

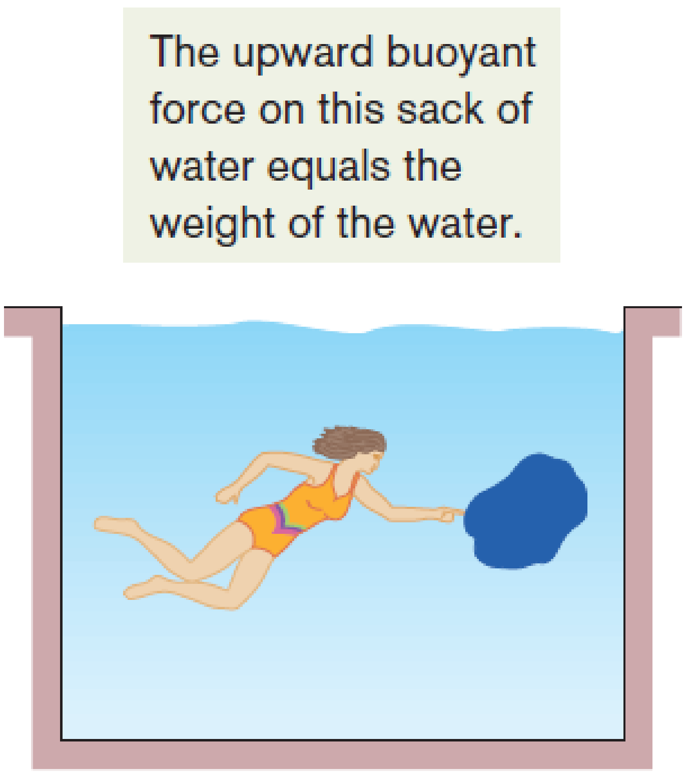


Fig. 14-9 A thin-walled plastic sack of water is in static equilibrium in the pool. The gravitational force on the sack must be balanced by a net upward force on it from the surrounding water.

The net upward force on the object is the buoyant force, $\mathbf{F_b}$.

The buoyant force on a body in a fluid has the magnitude:

$$F_b = m_f\, g \quad \textit{(buoyant force)},$$

where m_f is the mass of the fluid that is displaced by the body.

14.7: Archimedes Principle: Floating & Apparent Weight

- When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body:

$$F_b = F_g$$

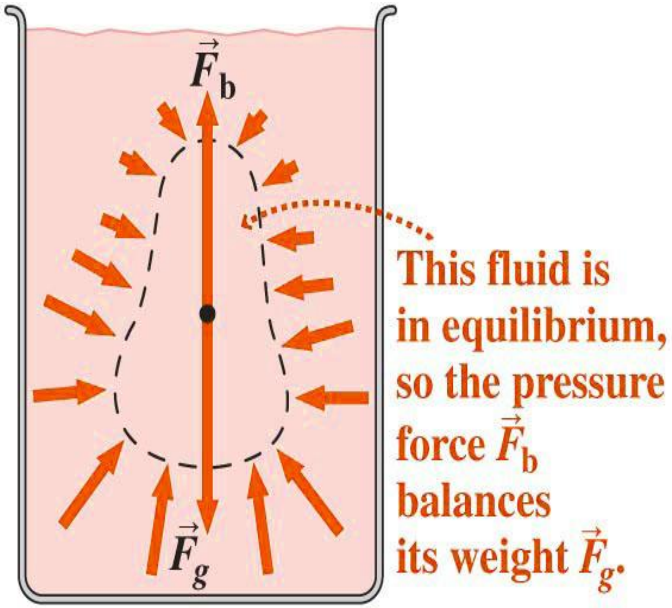
- When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f g$ of the fluid that has been displaced by the body, where m_f is the mass of the fluid displaced:

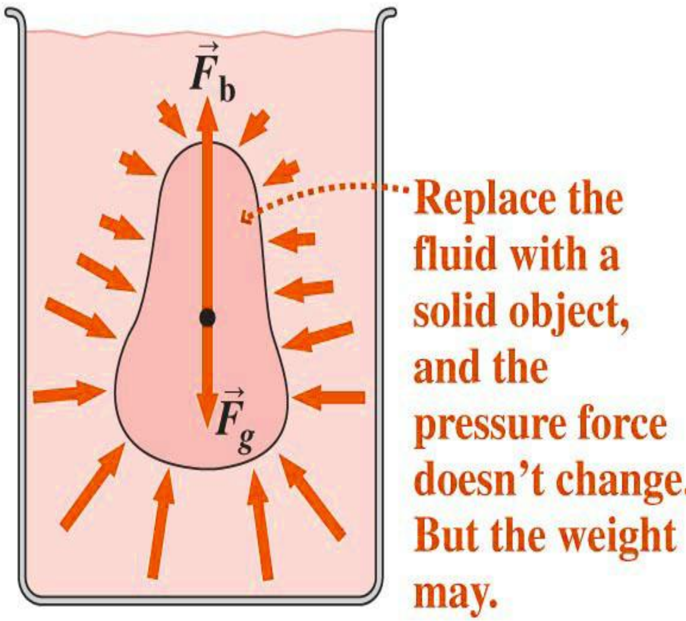
$$F_g = m_f g$$

- A floating body displaces its own weight of fluid.

- The apparent weight of an object in a fluid is less than the actual weight of the object in vacuum, and is equal to the difference between the actual weight of a body and the buoyant force on the body.

$$\begin{pmatrix}\text{apparent}\\ \text{weight}\end{pmatrix} = \begin{pmatrix}\text{actual}\\ \text{weight}\end{pmatrix} - \begin{pmatrix}\text{magnitude of}\\ \text{buoyant force}\end{pmatrix}$$

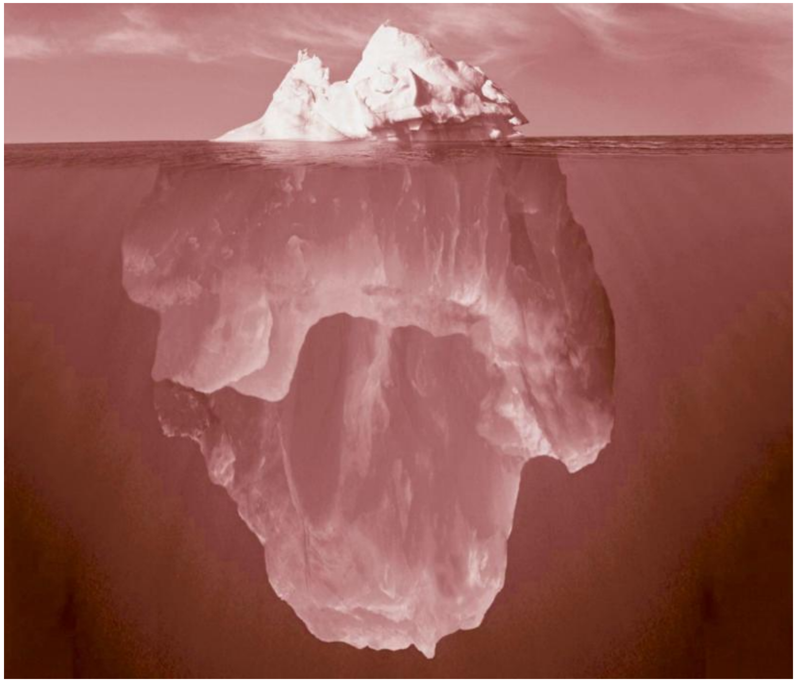




14.7: Archimedes Principle: Floating & Apparent Weight

Floating and sinking

- If a submerged object is less dense than a fluid, then the buoyancy force is greater than its weight, and the object rises.
  - In a liquid, it eventually reaches the surface.
    - Then the object floats at a level such that the buoyancy force equals its weight.
    - That means the submerged portion displaces a weight of liquid equal to the weight of the object.
  - In the atmosphere, a buoyant object like a balloon rises to a level where its density is equal to that of the atmosphere.
    - This is neutral buoyancy.



Clicker question

Which one of the following does not contribute to a rise in sea level?

A. Melting of pack ice in the Arctic Ocean
B. Warming of ocean water in the Pacific Ocean
C. Melting of ice supported by land in Greenland

Example: Floating, buoyancy, & density

In Fig. 14-11, a block of density $\rho = 800\ \text{kg/m}^3$ floats face down in a fluid of density $\rho_f = 1200\ \text{kg/m}^3$. The block has height $H = 6.0$ cm.

(a) By what depth h is the block submerged?

Floating means that the buoyant force matches the gravitational force.

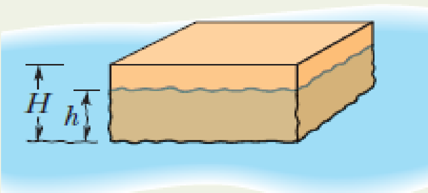


Fig. 14-11

$$F_b = m_f g = \rho_f V_f g = \rho_f LWhg.$$

$$F_g = mg = \rho V g = \rho_f LWHg.$$

$$F_b - F_g = m(0),$$

$$\rho_f LWhg - \rho LWHg = 0,$$

$$h = \frac{\rho}{\rho_f} H = \frac{800\ \text{kg/m}^3}{1200\ \text{kg/m}^3}(6.0\ \text{cm})$$
$$= 4.0\ \text{cm}.$$

(b) If the block is held fully submerged and then released, what is the magnitude of its acceleration?

$$F_b - F_g = ma,$$

$$\rho_f LWHg - \rho LWHg = \rho LWHa$$

$$a = \left(\frac{\rho_f}{\rho} - 1\right)g = \left(\frac{1200\ \text{kg/m}^3}{800\ \text{kg/m}^3} - 1\right)(9.8\ \text{m/s}^2)$$
$$= 4.9\ \text{m/s}^2. \qquad \text{(Answer)}$$

14.8: Ideal Fluids in Motion (Fluid Dynamics)

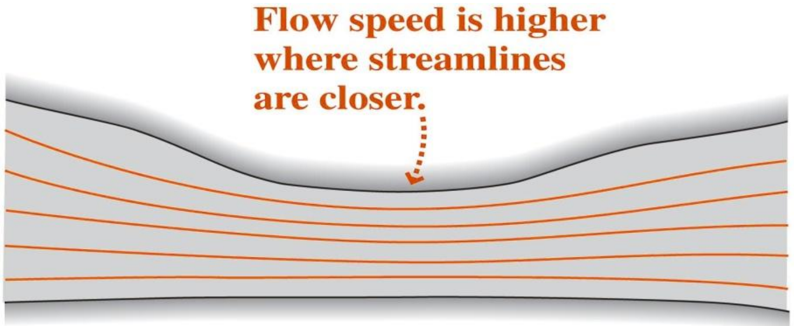
- ***Steady flow:*** In steady (or laminar) flow, the velocity of the moving fluid at any fixed point does not change with time.

- ***Incompressible flow: A***ssume, as for fluids at rest, that the ideal fluid is incompressible; density has a constant, uniform value.

- ***Nonviscous flow:*** The viscosity of a fluid is a measure of how resistive the fluid is to flow.
    - Viscosity is the fluid analog of friction between solids.
    - An object moving through a nonviscous fluid would experience no viscous drag force
    - No resistive force due to viscosity; can move at constant speed through the fluid.

- ***Irrotational flow:*** In irrotational flow a test body suspended in the fluid will not rotate about an axis through its own center of mass.

- Moving fluids are characterized by their flow velocity as a function of position and time.
    - In **steady flow**, the velocity at a given point is independent of time.
    - Steady flows can be visualized with **streamlines** which are everywhere tangent to the local flow direction.
    - The density of streamlines reflects the flow speed.



    - In **unsteady flow**, the fluid velocity at a given point varies with time.
    - Unsteady flows are more difficult to treat.

14.9: The Equation of Continuity

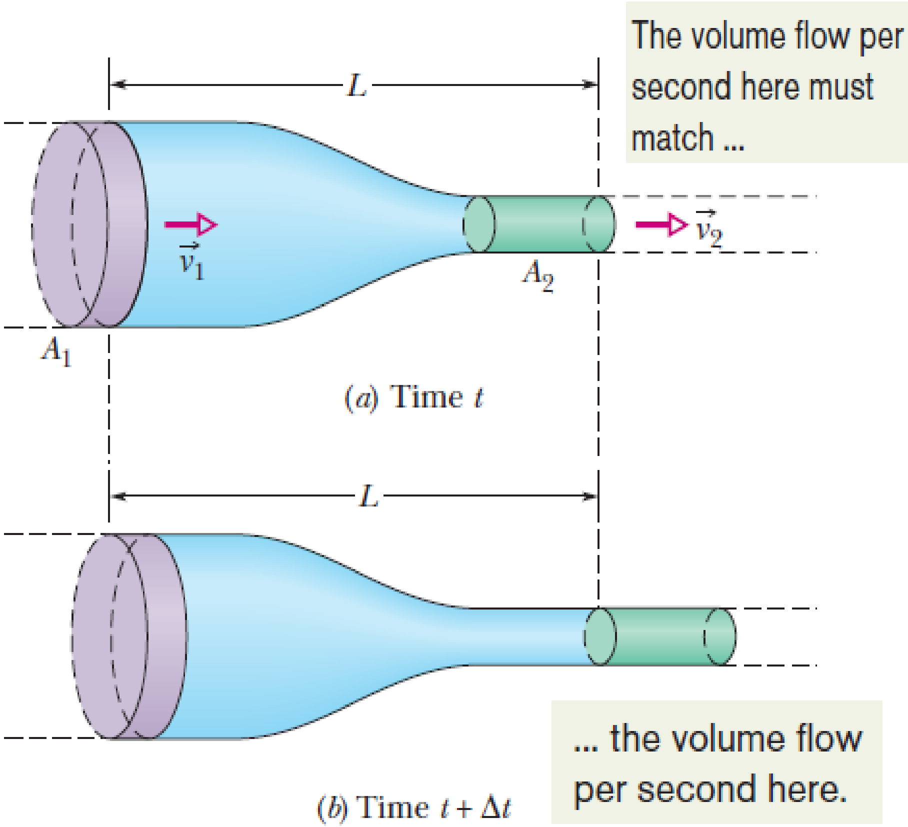


Fig. 14-15 Fluid flows from left to right at a steady rate through a tube segment of length L. The fluid's speed is v_1 at the left side and v_2 at the right side. The tube's cross-sectional area is A_1 at the left side and A_2 at the right side. From time t in (a) to time $t + \Delta t$ in (b), the amount of fluid shown in purple enters at the left side and the equal amount of fluid shown in green emerges at the right side.

$$\Delta V = A\,\Delta x = Av\,\Delta t.$$

$$\Delta V = A_1 v_1\,\Delta t = A_2 v_2\,\Delta t$$

$$A_1 v_1 = A_2 v_2 \qquad \text{(equation of continuity).}$$

Incompressible fluids (constant density):

$$\boxed{R_V = Av = \text{a constant}}$$

$$\boxed{R_m = \rho R_V = \rho A v = \text{a constant} \qquad \text{(mass flow rate).}}$$

Example: Water Stream

Figure 14-18 shows how the stream of water emerging from a faucet "necks down" as it falls. This change in the horizontal cross-sectional area is characteristic of any laminar (non-turbulant) falling stream because the gravitational force increases the speed of the stream. Here the indicated cross-sectional areas are $A_0 = 1.2\ \text{cm}^2$ and $A = 0.35\ \text{cm}^2$. The two levels are separated by a vertical distance $h = 45$ mm. What is the volume flow rate from the tap?

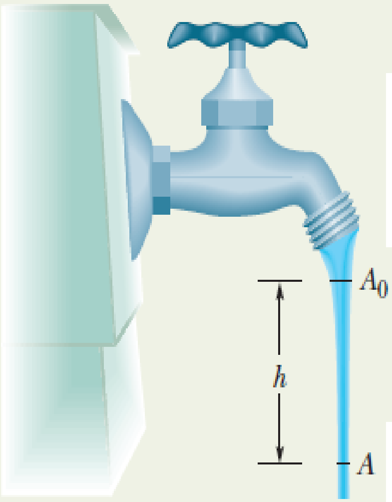


Fig. 14-18 As water falls from a tap, its speed increases. Because the volume flow rate must be the same at all horizontal cross sections of the stream, the stream must "neck down" (narrow).

KEY IDEA

The volume flow rate through the higher cross section must be the same as that through the lower cross section.

Calculations: From Eq. 14-24, we have

$$A_0 v_0 = Av, \tag{14-26}$$

where v_0 and v are the water speeds at the levels corresponding to A_0 and A. From Eq. 2-16 we can also write, because the water is falling freely with acceleration g,

$$v^2 = v_0^2 + 2gh. \tag{14-27}$$

Eliminating v between Eqs. 14-26 and 14-27 and solving for v_0, we obtain

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$= \sqrt{\frac{(2)(9.8\ \text{m/s}^2)(0.045\ \text{m})(0.35\ \text{cm}^2)^2}{(1.2\ \text{cm}^2)^2 - (0.35\ \text{cm}^2)^2}}$$

$$= 0.286\ \text{m/s} = 28.6\ \text{cm/s}.$$

From Eq. 14-24, the volume flow rate R_V is then

$$R_V = A_0 v_0 = (1.2\ \text{cm}^2)(28.6\ \text{cm/s})$$

$$= 34\ \text{cm}^3/\text{s}. \qquad \text{(Answer)}$$

14.10: Bernoulli's Equation

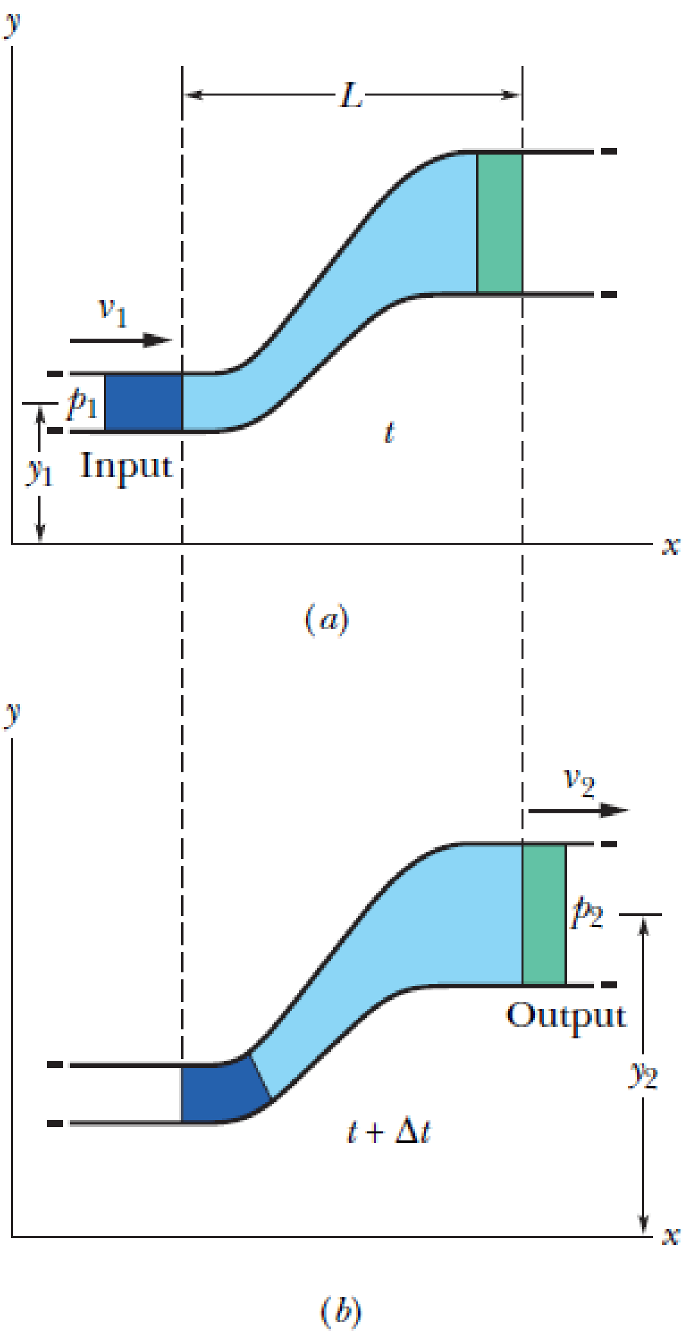


Fig. 14-19: Fluid flows at a steady rate through a length L of a flow tube, from the input end at the left to the output end at the right.

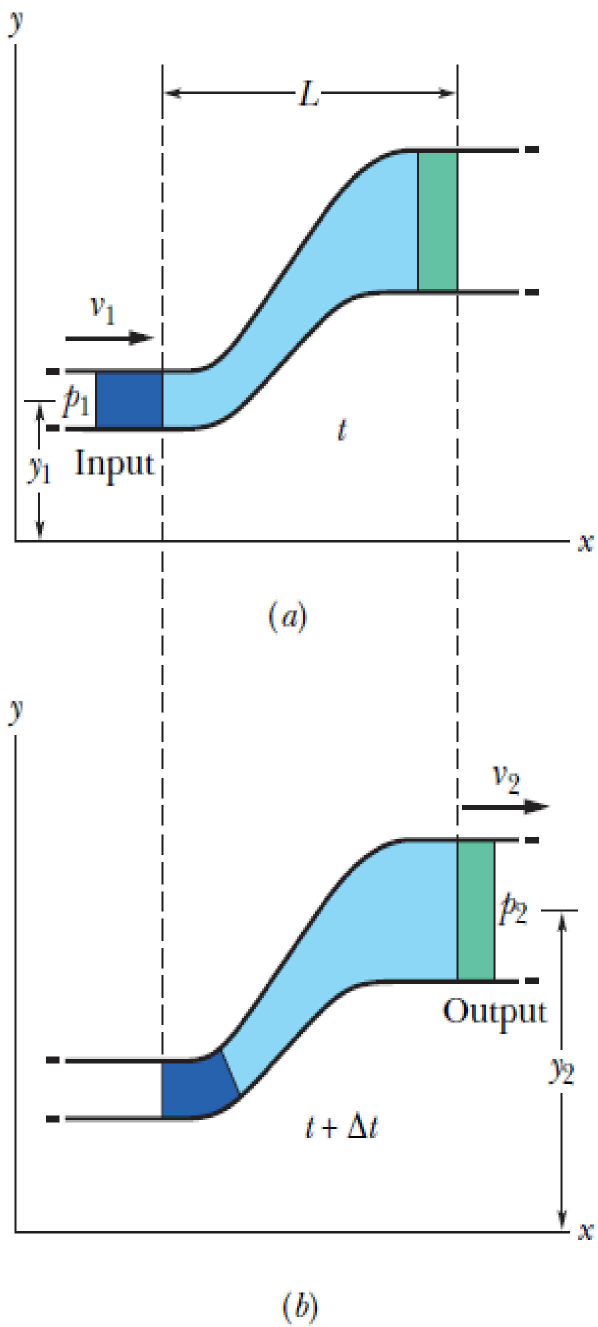
From time *t* in (a) to time *t* + *Δt* in (b), the amount of fluid shown in purple enters the input end and the equal amount shown in green emerges from the output end.

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant} \qquad \text{(Bernoulli's equation).}$$

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

14.10: Bernoulli's Equation: Proof



The change in kinetic energy of the system is equal to the work done on the system:

$$W = \Delta K,$$

If the density of the fluid is ρ,

$$\begin{aligned} \Delta K &= \tfrac{1}{2}\Delta m\, v_2^2 - \tfrac{1}{2}\Delta m\, v_1^2 \\ &= \tfrac{1}{2}\rho\, \Delta V (v_2^2 - v_1^2), \end{aligned}$$

The work done by gravitational forces is:

$$\begin{aligned} W_g &= -\Delta m\, g(y_2 - y_1) \\ &= -\rho g\, \Delta V (y_2 - y_1). \end{aligned}$$

The net work done by the fluid is:

$$\begin{aligned} W_p &= -p_2\, \Delta V + p_1\, \Delta V \\ &= -(p_2 - p_1)\, \Delta V. \end{aligned}$$

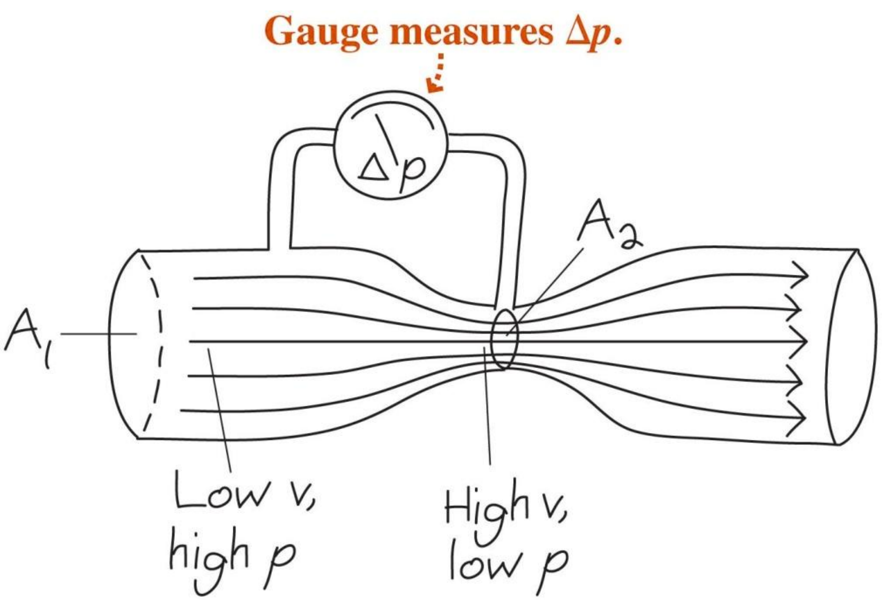
Therefore, $W = W_g + W_p = \Delta K.$

Finally, $-\rho g\, \Delta V (y_2 - y_1) - \Delta V (p_2 - p_1) = \tfrac{1}{2}\rho\, \Delta V (v_2^2 - v_1^2).$

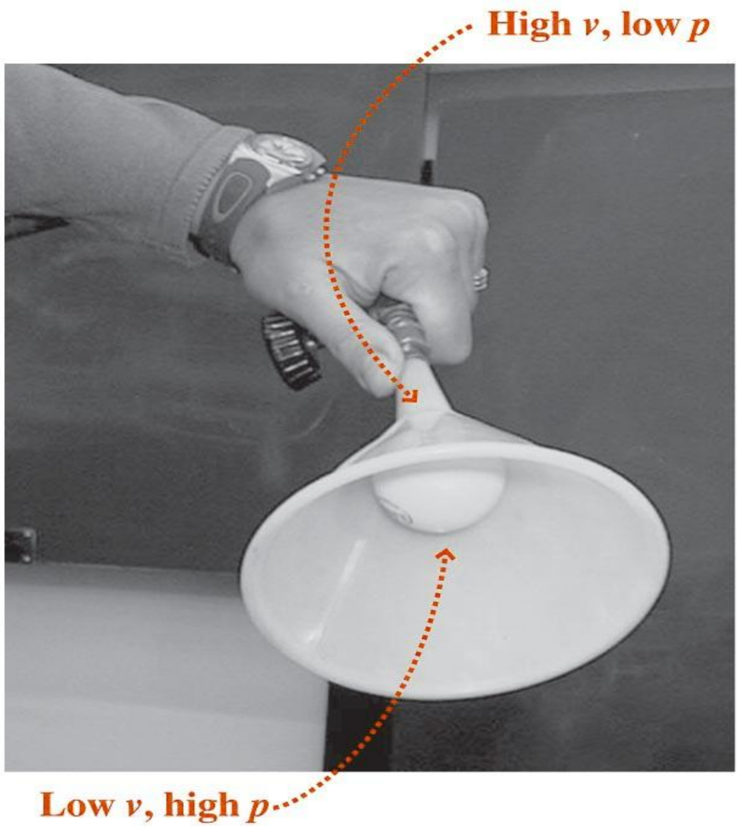
14.10: Bernoulli's Equation

The Bernoulli effect

- For flows that don't involve height differences, Bernoulli's equation shows that higher flow speeds are accompanied by lower pressures, and vice versa.



The venturi flow meter is one application of the Bernoulli effect. Measuring the pressure difference between the constriction and the unconstricted pipe gives a measure of the flow speed.

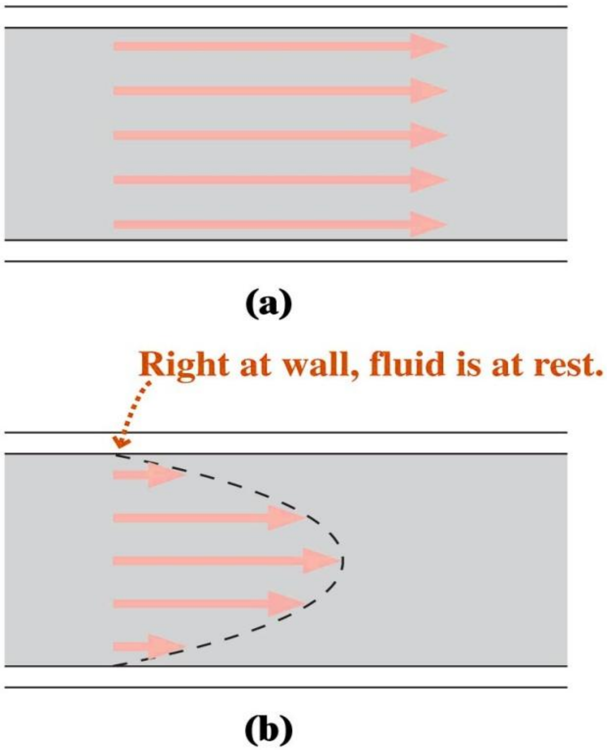


The ping-pong ball is supported by the downward flowing air in the inverted funnel, because of the higher pressure of the slower-moving air beneath the ball.

14.10: Bernoulli's Equation

Viscosity & turbulence

- **Viscosity**: fluid friction
  - Associated with the transfer of momentum by molecules moving perpendicular to the fluid flow
  - Also occurs where a fluid contacts pipe walls, river banks, and other material containers
  - Dissipates flow energy
- **Turbulence**: complex, chaotic, time-dependent fluid motion.



Without viscosity, flow in a pipe would be uniform. Viscous drag at the pipe walls introduces a parabolic flow profile.

Example: Bernoulli's Principle

Ethanol of density $\rho = 791\ \text{kg/m}^3$ flows smoothly through a horizontal pipe that tapers (as in Fig. 14-15) in cross-sectional area from $A_1 = 1.20 \times 10^{-3}\ \text{m}^2$ to $A_2 = A_1/2$. The pressure difference between the wide and narrow sections of pipe is 4120 Pa. What is the volume flow rate R_V of the ethanol?

KEY IDEAS

(1) Because the fluid flowing through the wide section of pipe must entirely pass through the narrow section, the volume flow rate R_V must be the same in the two sections. Thus, from Eq. 14-24,

$$R_V = v_1 A_1 = v_2 A_2. \tag{14-35}$$

However, with two unknown speeds, we cannot evaluate this equation for R_V. (2) Because the flow is smooth, we can apply Bernoulli's equation. From Eq. 14-28, we can write

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y, \tag{14-36}$$

where subscripts 1 and 2 refer to the wide and narrow sections of pipe, respectively, and y is their common elevation. This equation hardly seems to help because it does not contain the desired R_V and it contains the unknown speeds v_1 and v_2.

Calculations: There is a neat way to make Eq. 14-36 work for us: First, we can use Eq. 14-35 and the fact that $A_2 = A_1/2$ to write

$$v_1 = \frac{R_V}{A_1} \quad \text{and} \quad v_2 = \frac{R_V}{A_2} = \frac{2R_V}{A_1}. \tag{14-37}$$

Then we can substitute these expressions into Eq. 14-36 to eliminate the unknown speeds and introduce the desired volume flow rate. Doing this and solving for R_V yield

$$R_V = A_1 \sqrt{\frac{2(p_1 - p_2)}{3\rho}}. \tag{14-38}$$

We still have a decision to make: We know that the pressure difference between the two sections is 4120 Pa, but does that mean that $p_1 - p_2$ is 4120 Pa or -4120 Pa? We could guess the former is true, or otherwise the square root in Eq. 14-38 would give us an imaginary number. Instead of guessing, however, let's try some reasoning. From Eq. 14-35 we see that speed v_2 in the narrow section (small A_2) must be greater than speed v_1 in the wider section (larger A_1). Recall that if the speed of a fluid increases as the fluid travels along a horizontal path (as here), the pressure of the fluid must decrease. Thus, p_1 is greater than p_2, and $p_1 - p_2 =$ 4120 Pa. Inserting this and known data into Eq. 14-38 gives

$$R_V = 1.20 \times 10^{-3}\ \text{m}^2 \sqrt{\frac{(2)(4120\ \text{Pa})}{(3)(791\ \text{kg/m}^3)}}$$

$$= 2.24 \times 10^{-3}\ \text{m}^3/\text{s}. \qquad \text{(Answer)}$$

Example-2: Bernoulli's Principle

In the old West, a desperado fires a bullet into an open water tank (Fig. 14-20), creating a hole a distance h below the water surface. What is the speed v of the water exiting the tank?

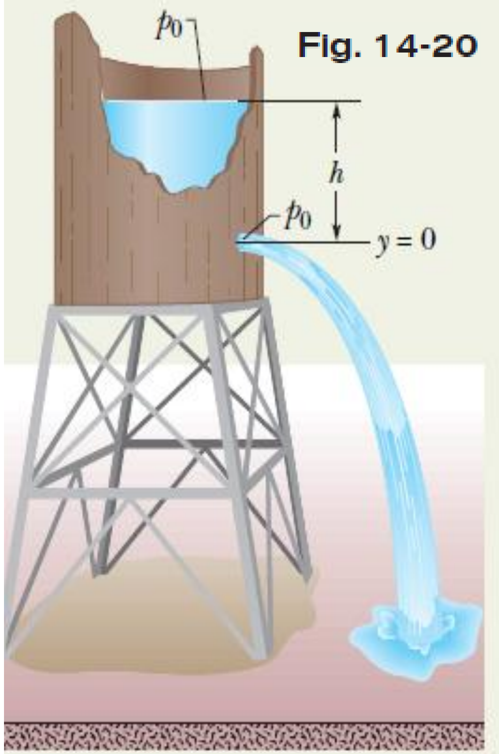


Fig. 14-20 Water pours through a hole in a water tank, at a distance h below the water surface. The pressure at the water surface and at the hole is atmospheric pressure p_0.

$$R_V = av = Av_0$$

and thus

$$v_0 = \frac{a}{A} v.$$

Because $a \ll A$, we see that $v_0 \ll v$. To apply Bernoulli's equation, we take the level of the hole as our reference level for measuring elevations (and thus gravitational potential energy). Noting that the pressure at the top of the tank and at the bullet hole is the atmospheric pressure p_0 (because both places are exposed to the atmosphere), we write Eq. 14-28 as

$$p_0 + \tfrac{1}{2}\rho v_0^2 + \rho g h = p_0 + \tfrac{1}{2}\rho v^2 + \rho g(0). \qquad (14\text{-}39)$$

(Here the top of the tank is represented by the left side of the equation and the hole by the right side. The zero on the right indicates that the hole is at our reference level.) Before we solve Eq. 14-39 for v, we can use our result that $v_0 \ll v$ to simplify it: We assume that v_0^2, and thus the term $\frac{1}{2}\rho v_0^2$ in Eq. 14-39, is negligible relative to the other terms, and we drop it. Solving the remaining equation for v then yields

$$v = \sqrt{2gh}. \qquad \text{(Answer)}$$

This is the same speed that an object would have when falling a height h from rest.